Cayley-type conditions for billiards within k quadrics in d

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Corrigendum

Cayley-type conditions for billiards within k quadrics in $\mathbb{R}^d$
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In section 4 of this paper, we misstated theorems 3 and 4. They should be reformulated as follows.

Theorem 3. *A trajectory of the billiard system constrained to the ellipsoid $\mathcal{E}$ within Ω : $\beta_1' \leqslant \lambda_1 \leqslant \beta_1'', \ldots, \beta_{d-1}' \leqslant \lambda_{d-1} \leqslant \beta_{d-1}''$, with caustics $Q_{\alpha_1}, \ldots, Q_{\alpha_{d-2}}$, is periodic with exactly n_s bounces at each of quadrics $Q_{\gamma_s'}, Q_{\gamma_s''}$ $(1 \leqslant s \leqslant d-2)$ if and only if*

$$\sum_{s=1}^{d-1} n_s\big(\bar{\mathcal{A}}(P_{\gamma_s'}) - \bar{\mathcal{A}}(P_{\gamma_s''})\big) = 0$$

on the Jacobian of the curve

$$\Gamma_1 \;:\; y^2 = \mathcal{P}_1(x) := -x(a_1 - x)\cdots(a_d - x)(\alpha_1 - x)\cdots(\alpha_{d-2} - x).$$

Here $P_{\gamma_s'}$, $P_{\gamma_s''}$ are the points on Γ_1 with coordinates $P_{\gamma_s'} = \big(\gamma_s', (-1)^s\sqrt{\mathcal{P}_1(\gamma_s')}\big)$, $P_{\beta_s''} = \big(\gamma_s'', (-1)^s\sqrt{\mathcal{P}_1(\gamma_s'')}\big)$, with $[\gamma_s', \gamma_s''] = \{\lambda \in [\beta_s', \beta_s''] \;:\; \mathcal{P}_1(\lambda) \geqslant 0\}$, $1 \leqslant s \leqslant d-2$, and $\bar{\mathcal{A}}(P) = \left(0, \int_0^P \frac{x\,dx}{y}, \int_0^P \frac{x^2dx}{y}, \ldots, \int_0^P \frac{x^{d-2}dx}{y}\right)$.

Associate with the billiard ordered game the following divisors on the curve Γ_1:

$$\mathcal{D}_s = \begin{cases} P_{\mu''} & \text{if } i_s = i_{s+1} = 1 \\ 0 & \text{if } i_s = -i_{s+1} = 1, \beta_s < \beta_{s+1} \text{ or } i_s = -i_{s+1} = -1, \beta_s > \beta_{s+1} \\ P_{\mu''} - P_{\mu'} & \text{if } i_s = -i_{s+1} = 1, \beta_s > \beta_{s+1} \\ P_{\mu'} - P_{\mu''} & \text{if } i_s = -i_{s+1} = -1, \beta_s < \beta_{s+1} \\ P_{\mu'} & \text{if } i_s = i_{s+1} = -1, \end{cases}$$

where $P_{\mu'}$ and $P_{\mu''}$ are its branching points with coordinates $(\mu', 0)$ and $(\mu'', 0)$, respectively.

Theorem 4. *Given a billiard ordered game constrained to $\mathcal{E}$ within quadrics $Q_{\beta_1}, \ldots, Q_{\beta_k}$ with signature $\sigma = (i_1, \ldots, i_k)$. Its trajectory with caustics $Q_{\alpha_1}, \ldots, Q_{\alpha_{d-2}}$ is k-periodic if and only if*

$$\sum_{s=1}^{k} i_s\big(\bar{\mathcal{A}}(P_{\beta_s}) - \bar{\mathcal{A}}(\mathcal{D}_s)\big)$$

is equal to a sum of several expressions of the form $\bar{\mathcal{A}}(P_{\alpha_p}) - \bar{\mathcal{A}}(P_{\alpha_{p'}})$ on the Jacobian of the curve $\Gamma_1 : y^2 = \mathcal{P}_1(x)$, where $P_{\beta_s} = \big(\beta_s, +\sqrt{\mathcal{P}_1(\beta_s)}\big)$ and Q_{α_p}, $Q_{\alpha_{p'}}$ are pairs of caustics of the same type.

Note, however, that the example given there (proposition 2) to illustrate these two theorems, is correct.

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