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Cayley-type conditions for billiards within k quadrics in d

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Corrigendum

Cayley-type conditions for billiards within k **quadrics in** \mathbb{R}^d

V Dragović and M Radnović 2004 J. Phys. A: Math. Gen. 37 1269-76

In section 4 of this paper, we misstated theorems 3 and 4. They should be reformulated as follows.

Theorem 3. A trajectory of the billiard system constrained to the ellipsoid \mathcal{E} within Ω : $\beta'_1 \leq \lambda_1 \leq \beta''_1, \ldots, \beta'_{d-1} \leq \lambda_{d-1} \leq \beta''_{d-1}$, with caustics $Q_{\alpha_1}, \ldots, Q_{\alpha_{d-2}}$, is periodic with exactly n_s bounces at each of quadrics $Q_{\gamma'_i}, Q_{\gamma''_i}$ $(1 \leq s \leq d-2)$ if and only if

$$\sum_{s=1}^{d-1} n_s \left(\bar{\mathcal{A}}(P_{\gamma'_s}) - \bar{\mathcal{A}}(P_{\gamma''_s}) \right) = 0$$

on the Jacobian of the curve

$$\Gamma_1 : y^2 = \mathcal{P}_1(x) := -x(a_1 - x) \cdots (a_d - x)(\alpha_1 - x) \cdots (\alpha_{d-2} - x).$$

Here $P_{\gamma'_s}$, $P_{\gamma''_s}$ are the points on Γ_1 with coordinates $P_{\gamma'_s} = (\gamma'_s, (-1)^s \sqrt{\mathcal{P}_1(\gamma'_s)})$, $P_{\beta''_s} = (\gamma''_s, (-1)^s \sqrt{\mathcal{P}_1(\gamma''_s)})$, with $[\gamma'_s, \gamma''_s] = \{\lambda \in [\beta'_s, \beta''_s] : \mathcal{P}_1(\lambda) \ge 0\}$, $1 \le s \le d-2$, and $\bar{\mathcal{A}}(P) = (0, \int_0^P \frac{xdx}{y}, \int_0^P \frac{x^2dx}{y}, \dots, \int_0^P \frac{x^{d-2}dx}{y})$.

Associate with the billiard ordered game the following divisors on the curve Γ_1 :

$$\mathcal{D}_{s} = \begin{cases} P_{\mu''} & \text{if } i_{s} = i_{s+1} = 1\\ 0 & \text{if } i_{s} = -i_{s+1} = 1, \, \beta_{s} < \beta_{s+1} \text{ or } i_{s} = -i_{s+1} = -1, \, \beta_{s} > \beta_{s+1}\\ P_{\mu''} - P_{\mu'} & \text{if } i_{s} = -i_{s+1} = 1, \, \beta_{s} > \beta_{s+1}\\ P_{\mu'} - P_{\mu''} & \text{if } i_{s} = -i_{s+1} = -1, \, \beta_{s} < \beta_{s+1}\\ P_{\mu'} & \text{if } i_{s} = i_{s+1} = -1, \end{cases}$$

where $P_{\mu'}$ and $P_{\mu''}$ are its branching points with coordinates $(\mu', 0)$ and $(\mu'', 0)$, respectively.

Theorem 4. Given a billiard ordered game constrained to \mathcal{E} within quadrics $Q_{\beta_1}, \ldots, Q_{\beta_k}$ with signature $\sigma = (i_1, \ldots, i_k)$. Its trajectory with caustics $Q_{\alpha_1}, \ldots, Q_{\alpha_{d-2}}$ is k-periodic if and only if

$$\sum_{s=1}^k i_s \big(\bar{\mathcal{A}}(P_{\beta_s}) - \bar{\mathcal{A}}(\mathcal{D}_s) \big)$$

is equal to a sum of several expressions of the form $\overline{A}(P_{\alpha_p}) - \overline{A}(P_{\alpha_{p'}})$ on the Jacobian of the curve $\Gamma_1 : y^2 = \mathcal{P}_1(x)$, where $P_{\beta_s} = (\beta_s, +\sqrt{\mathcal{P}_1(\beta_s)})$ and Q_{α_p} , $Q_{\alpha_{p'}}$ are pairs of caustics of the same type.

Note, however, that the example given there (proposition 2) to illustrate these two theorems, is correct.

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